Pergamon

Int. J. Heat Mass Transfer. Vol. 37, No. 13, p. 1969, 1994
Copyright © 1994 Elsevier Science Ltd
Printed in Great Britain. All rights reserved
0017-9310/94 $7.00+0.00

LETTER TO THE EDITORS

Comments on "Heat and mass transfer with a boundary layer flow past a flat plate of finite thickness"

IN THEIR paper published in this journal [1], Mori *et al.* theoretically analyzed heat and mass transfer in evaporation from a flat-plate surface to a laminar boundary flow past the plate, taking into account the two-dimensional thermal conduction in the plate of finite thickness. Conjugation of the problem of external flow with the problem of heat transfer was achieved by solving the system of equations (1)–(13) for the boundary layer and equation (15) with parametric boundary conditions

$$\hat{Y} = 0: \quad \theta_f = \theta_t(x^*). \tag{1*}$$

The authors solved these equations with boundary conditions (1*) and found the form of the parameter-function $\theta_i(X^*)$ from the condition of the equality of heat fluxes on the conjugate surface $\hat{y} = 0$. The solution of Laplace equation (15) for the plate of finite thickness with boundary condition (1*) is given in the form of functional series (55)

$$\theta_w(x^*, \hat{y}) = \frac{1+Bi(1+\hat{y})}{1+Bi} \cdot \int_0^1 \theta_i(\xi) \cdot \mathrm{d}\xi + 2\sum_{k=1}^{\infty} \frac{k\pi L^* \cdot \mathrm{ch}\{k\pi L^*(1+\hat{y})\} + Bi \cdot \mathrm{sh}\{k\pi L^* \cdot (1+y^*)\}}{k\pi L^* \cdot \mathrm{ch}\{k\pi L^*\} + Bi \cdot \mathrm{sh}\{k\pi L^*\}} \cdot \cos(k\pi x^*) \int_0^1 \theta_i(\xi) \cdot \cos(k\pi\xi)\,\mathrm{d}\xi. \tag{2*}$$

Being the result of solution, this series converges uniformly in $(x^*, \hat{y})$ at every point of the compact set $0 \geqslant x^* \geqslant 1$ and $-1 \leqslant \hat{y} \leqslant 0$. So, solution (2*) is a continuous function of the independent variables $(x^*, \hat{y})$. To realize the second boundary condition of conjugation (19) on the side of the plate, it is necessary to find the derivative $\partial\theta_w/\partial y|_{\hat{y}=0}$. To find this derivative, the authors of ref. [1] differentiated series (2*) termwise but this is incorrect mathematically since the series consists of derivatives diverging in $x^* = 0$ and $x^* = 1$ (harmonic series). Their results are correct physically, since sufficiently smooth distributions of the gas parameters on the outer edge of the boundary layer u_∞, T_∞ and the constant temperature of the liquid $T_h = \text{const.}$ make the problem absolutely stable for error (pointwise) when determining heat flux in the plate. But, in the case of critical regimes of heat transfer on the plate surface $\hat{y} = 0$, this technique may yield incorrect physical results. To avoid this, the following method of the conjugation of equations (1)–(13) and (15) can be used. First, we must solve these equations with the boundary conditions

$$\hat{y} = 0: \quad \frac{\partial\theta_w}{\partial\hat{y}} = -q_i^*(x^*) = -\frac{\delta}{k_w(T_\infty - T_h)} \cdot q_i(x^*) \tag{3*}$$

where $q_i^*(x^*)$ is the parametric function of the heat flux on the surface $\hat{y} = 0$. The solution of equation (15) with boundary conditions (3*) takes on the form

$$\theta_w(x^*, \hat{y}) = -\frac{1}{Bi} \cdot \int_0^1 q_i^*(\xi)\,\mathrm{d}\xi - 2 \cdot \sum_{k=1}^{\infty} \frac{k\pi L^* \cdot \mathrm{ch}\{k\pi L^*(1+\hat{y})\} + Bi \cdot \mathrm{sh}\{k\pi L^*(1+\hat{y})\}}{k\pi L^* \cdot \mathrm{sh}\{k\pi L^*\} + Bi \cdot k\pi L^* \cdot \mathrm{ch}\{k\pi L^*\}} \cdot \cos(k\pi x^*) \int_0^1 q_i^*(\xi) \cdot \cos(k\pi\xi)\,\mathrm{d}\xi. \tag{4*}$$

Functional series (4*) converges uniformly over the whole domain: $0 \leqslant x^* \leqslant 1$ and $-1 \leqslant \hat{y} \leqslant 0$. Conjugation of the solutions of equations (1)–(13) and (15) with boundary conditions (3*) is achieved by using the temperature equation

$$\hat{y} = 0: \quad \theta_f(x^*) = \theta_w(x^*, 0). \tag{5}$$

As the method does not need the derivative of function series (4*), there are no troubles with the divergence in boundary points.

Remark: (N), labels in ref. [1]
(N^*), labels used in this letter.

ANDREY G. FEDOROV
Moscow Bauman State Technical University,
Department of Energy Machine Building,
2-nd Baumanskaya Str., 5
Moscow, 107005, *Russia*

REFERENCE

1. S. Mori, H. Nakagawa, A. Tanimoto and M. Sakakibara, Heat and mass transfer with a boundary layer flow past a flat plate of finite thickness, *Int. J. Heat Mass Transfer* **34**, 2899–2909 (1991).